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Model for Nearfield Electromagnetic Shielding by Cylindrical Shells of Composite Materials

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PREFACE

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This report describes the shielding of sources by cylindrical shells of composite materials that are characterized by complex permeability and permittivity. The primary objective of the investigation is to develop a mathematical model of the magnetic, magnetizing, electric, and displacement fields as they exist in the presence of the cylindrical shell. This is accomplished by solving the magnetostatic problem, first by formulating the nearfield source in binomial expansions and the fields around the cylindrical shell in powers of Legendre polynomials and then by applying the boundary conditions of the fields to the cylindrical surface of the shell. The magnetostatic solution is used as input to alternate iterations of Faraday's law and Ampere's law to generate a perturbation expansion of the fields.

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MODEL FOR NEARFIELD ELECTROMAGNETIC SHIELDING BY CYLINDRICAL SHELLS OF COMPOSITE MATERIALS

1. INTRODUCTION

This investigation focuses on the shielding of sources at low frequencies by cylindrical shells of composite materials characterized by complex permeability and permittivity. In brief, the magnetostatic problem is solved by first formulating the nearfield source in binomial expansions and the fields around the cylindrical shell in powers of Legendre polynomials.[1-5] Next, the boundary conditions of the fields are applied to the cylindrical surface. The magnetostatic solution is then used as input to alternate iterations of Faraday's law and Ampere's law to generate a perturbation expression of the fields. The primary objective of this process is to develop a mathematical model of the H, B, E, and D fields* as they exist in the presence of the cylindrical shell.

More specifically, the time-averaged Poynting vector must be obtained. This requires solution of Maxwell's equations by iteration, and begins with calculations of the *B* and *H* fields in the magnetostatic limit for source models representing transformers and power supplies near the cylindrical shell. The boundary conditions for the continuity of the tangential component of the *H* field and the normal component of the *B* field are applied at the two interfaces of the cylindrical shell, as shown in figure 1.

The resulting magnetostatic fields are then substituted into Ampere's circuital law by equating the curl of the *H* field to the current density and the time derivative of the displacement vector. The *E* field can be extracted from the current density by using Ohm's law. The solution for the *E* field is generated from and, by iteration, is substituted into Faraday's law to yield improved *H* and *B* fields. The iteration proceeds back and forth between Ampere's law and Faraday's law, generating an infinite series of *H* and *B* correction terms, as described by Feynman.[6] This series of *E* and *H* fields (truncated for the purpose of this investigation) will yield the *E* and *H* fields used in the calculation of the Poynting vectors. Because this is a nearfield model, retarded time effects can be ignored.

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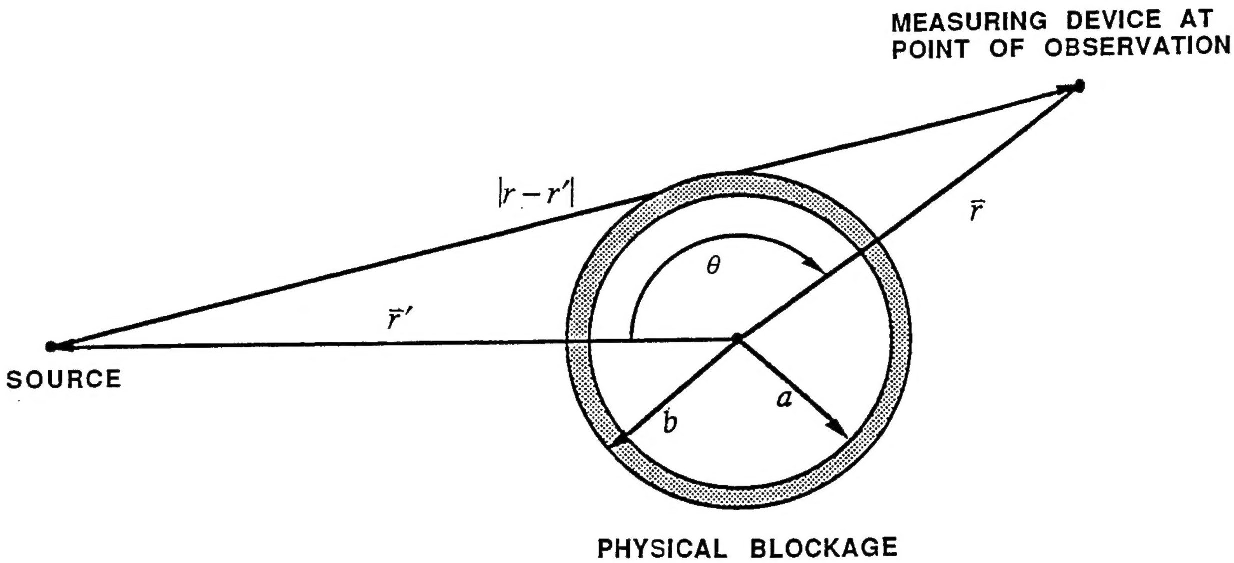
* According to Feynman's definition, *B* represents the magnetic field vector and *H* represents the magnetizing field. The term E represents the electric field, and D is the displacement field.



Note: The power supply field varies as $1/|\vec{r} - \vec{r}'|$ and the transformer field varies as $1/|\vec{r} - \vec{r}'|^3$, where the term $\vec{r}'$ is the distance from the source to the center of the cylindrical shell and $\vec{r}$ is the distance from the cylindrical axis to the measuring device at the point of observation. The coordinates at the point of observation are always $\vec{r}$ and θ.

Figure 1. Representation of the Cylindrical Shell (or Blockage)

2. THE MATHEMATICAL MODEL

As described in section 1, the model is based on an iteration process between Ampere's law

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} ,$$

where $\vec{J}$ is the current density and D is the displacement field vector, and Faraday's law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} ,$$

where $\nabla \times$ is the curl of the E field and t is time. The $\vec{H}$ field is related to $\vec{B}$ by $\vec{B} = \mu \vec{H}$, where the permeability μ is complex for a composite material. The displacement field $\vec{D}$ is related to $\vec{E}$ by $D = \varepsilon \vec{E}$, where E is complex for a composite material.

Ohm's law is used to express the current density in terms of the E field as

$$\vec{J} = \sigma \vec{E} ,$$

where σ is the conductivity.

We use the complex amplitudes of the fields,

$$\vec{E} = \mathrm{Re}\, \hat{\vec{E}}^{e^{jwt}} ,$$

where Re represents the real part and $\hat{\vec{E}}$ is the complex amplitude of the electric field, to generate the Poynting vector

$$\vec{S} = \frac{1}{2} \mathrm{Re}\left(\hat{\vec{E}} \times \hat{\vec{H}}^{*} \right) .$$

The Poynting vector is then used to derive field intensities at the point of observation:

$$\vec{H} = \mathrm{Re}\, \hat{\vec{H}}^{e^{jwt}} .$$

The iteration process can be truncated after only a few iterations owing to the presence of low frequencies. In fact, for very low frequencies, we may only need Ampere's law to extract the E field from the H field (which has been derived from the magnetostatic model).

3. THE POWER SUPPLY SOURCE IN THE PRESENCE OF A CYLINDRICAL SHELL OF COMPOSITE MATERIAL IN THE MAGNETOSTATIC LIMIT

THE MATHEMATICAL FORM FOR THE $1/|\bar{r}-\bar{r}'|$ SOURCE

For the power supply source, the H fields are given in three regions: $r > b$, $a < r < b$, and $r < a$, where a and b are the inner and outer radii of the cylindrical shell, respectively.

For $r > b$, the H field is given by

$$\bar{H} = \frac{B_o}{|\bar{r}-\bar{r}'|}\frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|} - \nabla\sum_{\ell=0}^{2}\frac{\alpha_\ell}{r^{\ell+1}}P_\ell\left[\cos(\theta)\right], \tag{1}$$

where $P_\ell(\cos\theta)$ are the Legendre polynomials.

For $a < r < b$,

$$\bar{H} = -\nabla\sum_{\ell=0}^{2}\left\{\beta_\ell r^\ell + \frac{\gamma_\ell}{r^{\ell+1}}P_\ell\left[\cos(\theta)\right]\right\}, \tag{2}$$

and for $r < a$,

$$H = -\nabla\sum_{\ell=0}^{2}\delta_\ell r^\ell P_\ell\left[\cos(\theta)\right]. \tag{3}$$

THE $1/|\bar{r}-\bar{r}'|$ SOURCE IN COMPONENT FORM AND ITS BINOMIAL EXPANSION

The B field for the $1/|\bar{r}-\bar{r}'|$ can be expressed as

$$B = \frac{B_o}{|\bar{r}-\bar{r}'|}\frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|} \ ,$$

where the second factor is the unit vector extending from the source to the observation point. Taking the dot product of B with the unit vectors in the radial and angular directions of the physical blockage cylindrical coordinate system yields

$$B_r = \frac{B_o}{|\vec{r}-\vec{r}'|}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|} \bullet \frac{\vec{r}}{r} = \frac{B_o}{|\vec{r}-\vec{r}'|^2}(r - r'\cos\theta) \tag{4}$$

and

$$B_\theta = \frac{B_o}{|\vec{r}-\vec{r}'|^2}(\vec{r}-\vec{r}') \bullet \theta = B_o \frac{r'\sin\theta}{|\vec{r}-\vec{r}'|^2} \ , \tag{5}$$

where B_o is the source strength.

Expanding $1/|\vec{r}-\vec{r}'|^2$ in the binomial expansion yields

$$1\Big/|\vec{r}-\vec{r}'|^2 \approx \frac{1}{r^2}\left(1 - \frac{r'^2}{r^2} + \frac{2r'}{r}\cos\theta\right) \quad \text{for } \vec{r} \geq \vec{r}' \tag{6}$$

and

$$1\Big/|\vec{r}-\vec{r}'|^2 \approx \frac{1}{r'^2}\left(1 - \frac{r^2}{r'^2} + \frac{2r}{r'}\cos\theta\right) \quad \text{for } \vec{r} \leq \vec{r}'. \tag{7}$$

THE MATHEMATICAL SOLUTION OF THE B AND H FIELDS FOR THE $1/|\vec{r}-\vec{r}'|$ SOURCE

We substitute equations (6) and (7) into equations (4) and (5) and the result into the source terms of equations (1), (2), and (3). By applying the boundary conditions for the B and H fields to the resulting equations and setting the coefficients of the powers of the sines and cosines equal to zero, we generate 10 equations and 10 unknowns, which are solved to obtain the coefficients of expansion with the *Mathematica*[7] software (see appendix).

The boundary at $r = b$ has two cases: $\vec{r} \leq \vec{r}'$ and $\vec{r} \geq \vec{r}'$. For the continuity of B_r, the coefficients of $(\cos\theta)^0$ are

$$\frac{B_o}{r'^2}\left(b - \frac{b^3}{r'^2}\right) = \alpha_0 \frac{1}{b^2} - \frac{3}{2}\alpha_2 \frac{1}{b^4} - \mu\gamma_0 \frac{1}{b^2} - \mu\beta_1 - \mu\beta_2 b + \frac{3}{2}\mu \frac{\gamma_2}{b^4} \quad \text{for } \vec{r} \leq \vec{r}' \tag{8}$$

and

$$\frac{B_o}{b^2}\left(b-\frac{r'^2}{b}\right)=\alpha_0\frac{1}{b^2}-\frac{3}{2}\alpha_2\frac{1}{b^4}-\mu\gamma_0\frac{1}{b^2}-\mu\beta_1-\mu\beta_2 b+\frac{3}{2}\mu\frac{\gamma_2}{b^4}\quad \text{for } \bar{r}\geq\bar{r}'; \tag{9}$$

the coefficients of $(\cos\theta)^1$ are

$$\frac{B_0}{r'^2}\left(\frac{3b^2}{r'}-r'\right)=-2\alpha_1\frac{1}{b^3}-2\mu\gamma_1\frac{1}{b^3}\quad \text{for } \bar{r}\leq\bar{r}' \tag{10}$$

and

$$\frac{B_0}{b^2}\left(\frac{r'^3}{b^2}+r'\right)=-2\alpha_1\frac{1}{b^3}-2\mu\gamma_1\frac{1}{b^3}\quad \text{for } \bar{r}\geq\bar{r}'; \tag{11}$$

and the coefficients of $(\cos\theta)^2$ are

$$\frac{-2bB_o}{r'^2}=\frac{9}{2}\alpha_2\frac{1}{r^4}+3\mu\beta_2 b-\frac{9}{2}\mu\frac{\gamma_2}{b^4}\quad \text{for } \bar{r}\leq\bar{r}' \tag{12}$$

and

$$\frac{-2r'^2B_o}{b^3}=\frac{9}{2}\alpha_2\frac{1}{r^4}+3\mu\beta_2 b-\frac{9}{2}\mu\frac{\gamma_2}{b^4}\quad \text{for } \bar{r}\geq\bar{r}'. \tag{13}$$

For the continuity of H_θ at $r = b$, there are no terms in the coefficient for $(\cos\theta)^0$. For the coefficient of $(\sin)^1$,

$$\frac{B_0}{r'}\left(1-\frac{b^2}{r'^2}\right)-\frac{\alpha_1}{b^3}=-\beta_1-\frac{\gamma_1}{b^3}\quad \text{for } \bar{r}\leq\bar{r}' \tag{14}$$

and

$$\frac{B_0}{b^2}\left(r'-\frac{r'^3}{b^2}\right)-\frac{\alpha_1}{b^3}=-\beta_1-\frac{\gamma_1}{b^3}\quad \text{for } \bar{r}\geq\bar{r}'. \tag{15}$$

For the coefficient of $(\sin\theta\cos\theta)^1$,

$$\frac{2B_0 b}{r'^2}+\frac{3\alpha_2}{b^3}=3\beta_2 b^2+\frac{3\gamma_2}{b^4}\quad \text{for } \bar{r}\leq\bar{r}' \tag{16}$$

and

$$\frac{2B_0 r'^2}{b^3} + \frac{3\alpha_2}{b^3} = 3\beta_2 b^2 + \frac{3\gamma_2}{b^4} \quad \text{for} \quad \bar{r} . \tag{17}$$

For the continuity of B_r at $r = a$, the coefficient of $(\cos\theta)^0$ is

$$\frac{\mu\gamma_0}{a^2} - \mu\beta_2 a + \frac{3}{2}\frac{\gamma_2}{a^4} = -\frac{\delta_2 a}{2} ; \tag{18}$$

the coefficient of $(\cos\theta)^1$ is

$$\mu\left(\beta_1 - \frac{2\gamma_1}{a^3}\right) = \delta_1 ; \tag{19}$$

and the coefficient of $(\cos\theta)^2$ is

$$\mu\left(3\beta_2 a - \frac{9}{2}\frac{\gamma_2}{a^4}\right) = \frac{3}{2}\delta_2 a . \tag{20}$$

For the continuity of H_θ at $r = a$, the coefficient of $(\sin)^1$ is

$$\beta_1 a + \gamma_1 \frac{1}{a^2} = \delta_1 a , \tag{21}$$

and the coefficient of $(\sin\theta\cos\theta)^1$ is

$$\beta_2 a^2 + \gamma_2 \frac{1}{a^3} = \delta_2 a^2 . \tag{22}$$

The radii a and b are fixed known numbers, and the unknowns are

$\alpha_0, \alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_0, \gamma_1, \gamma_2, \delta_1,$ and δ_2 ,

with the following solutions:

$$\alpha_o = \frac{B_o\left[X_1\left(6\mu a^2 b - 12\mu^2 a^5 b + 6a^3 b^2 + 18\mu a^3 b^2 + 12\mu^2 a^3 b^2 + 4\mu b^5 + 6\mu^2 b^5\right)\right.}{6X_2X_4r'^4}$$

$$+\frac{\left.-6b^3X_2X_3 - 3\mu X_2X_5r' - 6\mu^2 X_2X_5r' - 6bX_2X_3r'^2 + 6b^4X_4X_6r'^2\right]}{6X_2X_4r'^4},$$

$$\alpha_1 = \frac{B_o\left(-3b^2 + r'^2\right)}{2r'^3}, \qquad \alpha_2 = \frac{2B_o b^4 X_6}{3X_2 r'^2},$$

$$\delta_1 = \frac{-3B_o \mu X_5}{2X_2 r'^2}, \qquad \delta_2 = \frac{10 B_o \mu b^4 X_7}{3X_2 r'^2},$$

which were solved numerically using the *Mathematica*[7] software (see appendix).

Substituting the above Legendre polynomial coefficients back into equations (1), (2), and (3) yields for $r > b$

$$\vec{H} = \hat{r}\left[\frac{B_0}{r^2 + r'^2 - 2rr'}(r - r'\cos\theta) - \frac{\alpha_1}{r^2}\cos\theta + \frac{\alpha_0}{r^2} + \frac{3\alpha_2}{2r^4}\left(3\cos^2\theta - 1\right)\right]$$

$$+\hat{\theta}\left[\frac{B_0 r'\sin\theta}{r^2 + r'^2 - 2rr'} - \frac{\alpha_1}{r}\sin\theta + \frac{\alpha_2}{2r^4}(6\cos\theta\sin\theta)\right];$$

for $b > r > a$

$$\vec{H} = \hat{r}\left[\left(\gamma_1\frac{2}{r^3} - \beta_1\right)\cos\theta + \frac{\gamma_0}{r^2} + \left(\gamma_2\frac{3}{r^4} + 2\beta_2 r\right)\left(3\cos^2\theta - 1\right)\right]$$

$$+\hat{\theta}\left[\left(\beta_1 + \frac{\gamma_1}{r^3}\right)\sin\theta - \beta_2 r + \frac{6\gamma_2}{r^4}\sin\theta\cos\theta\right];$$

and for $r < b$

$$\vec{H} = \hat{r}\left[\frac{2\delta_1}{r^3}\cos\theta - \frac{3\delta_2}{4r^4}\left(3\cos^2\theta - 1\right) + \frac{\delta_0}{r^2}\right] + \hat{\theta}\left(\frac{\delta_1}{r^3}\sin\theta + \frac{3\delta_2}{r^4}\sin\theta\cos\theta\right).$$

These equations have complex permeability.

4. THE TRANSFORMER SOURCE IN THE PRESENCE OF A CYLINDRICAL SHELL OF COMPOSITE MATERIAL IN THE MAGNETOSTATIC LIMIT

THE MATHEMATICAL FORM FOR THE $1/|\bar{r}-\bar{r}'|^3$ SOURCE

For the transformer source, the H fields are given in three regions: $r>b$, $a<r<b$, and $r<a$, where a and b are the inner and outer radii of the cylindrical shell, respectively. For $r > b$, the H field is given by

$$\bar{H} = \frac{B_0}{|\bar{r}-\bar{r}'|^3}\frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|} - \nabla\sum_{\ell=0}^{2}\frac{\alpha_\ell}{r^{\ell+1}}P_\ell[\cos(\theta)]\,; \tag{23}$$

for $a<r<b$,

$$\bar{H} = -\nabla\sum_{\ell=0}^{2}\left\{\beta_\ell r^\ell + \frac{\gamma_\ell}{r^{\ell+1}}P_\ell[\cos(\theta)]\right\}\,; \tag{24}$$

and for $r<a$,

$$\bar{H} = -\nabla\sum_{\ell=0}^{2}\delta_\ell r^\ell P_\ell[\cos(\theta)]\,. \tag{25}$$

Expanding equations (23), (24), and (25) yields for $r > b$

$$\bar{H} = \hat{r}\left[\frac{B_o}{\left(r^2+r'^2-2rr'\right)^2}(r-r'\cos\theta) - \frac{\alpha_1}{r^2}\cos\theta + \frac{\alpha_0}{r^2} + \frac{3\alpha_2}{2r^4}\left(3\cos^2\theta-1\right)\right]$$

$$+\hat{\theta}\left[\frac{B_o r'\sin\theta}{\left(r^2+r'^2-2rr'\right)^2} - \frac{\alpha_1}{r}\sin\theta + \frac{\alpha_2}{2r^4}(6\cos\theta\sin\theta)\right];$$

for $b>r>a$

$$\vec{H} = \hat{r}\left[\left(\gamma_1 \frac{2}{r^3} - \beta_1\right)\cos\theta + \frac{\gamma_0}{r^2} + \left(\gamma_2 \frac{3}{r^4} + 2\beta_2 r\right)\left(3\cos^2\theta - 1\right)\right]$$

$$+ \hat{\theta}\left[\left(\beta_1 + \frac{\gamma_1}{r^3}\right)\sin\theta - \beta_2 r + \frac{6\gamma_2}{r^4}\sin\theta\cos\theta\right];$$

and for $r < b$

$$\vec{H} = \hat{r}\left[\frac{2\delta_1}{r^3}\cos\theta - \frac{3\delta_2}{4r^4}(3\cos^2\theta - 1) + \frac{\delta_o}{r^2}\right]$$

$$+ \hat{\theta}\left[\frac{\delta_1}{r^3}\sin\theta + \frac{3\delta_2}{r^4}\sin\theta\cos\theta\right].$$

THE $1/|\bar{r} - \bar{r}'|^3$ SOURCE IN COMPONENT FORM AND ITS BINOMIAL EXPANSION

The B field for the $1/|\bar{r} - \bar{r}'|^3$ source can be expressed as

$$B = \frac{B_o}{|\bar{r} - \bar{r}'|^3} \frac{(\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|},$$

where the second factor is the unit vector extending from the source to the observation point. Taking the dot product of B with the unit vectors in the radial and angular directions of the physical blockage cylindrical coordinate system yields

$$B_r = \frac{B_o}{|\bar{r} - \bar{r}'|^3} \frac{(\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|} \bullet \frac{\bar{r}}{r} = \frac{B_o}{|\bar{r} - \bar{r}'|^4}(r - r'\cos\theta) \tag{26}$$

and

$$B_\theta = \frac{B_o}{|\bar{r} - \bar{r}'|^4}(\bar{r} - \bar{r}') \bullet \theta = B_o \frac{r'\sin\theta}{|\bar{r} - \bar{r}'|^4}. \tag{27}$$

Expanding $1/|\bar{r} - \bar{r}'|^4$ in the binomial expansion yields

$$1 \Big/ |\bar{r} - \bar{r}'|^4 \approx \frac{1}{r^4}\left[1 - 2\left(\frac{r'^2}{r^2} - \frac{2r'}{r}\cos\theta\right)\right] \quad \text{for} \quad \bar{r} \geq \bar{r}' \tag{28}$$

and

$$1 \Big/ |\vec{r}-\vec{r}'|^4 \approx \frac{1}{r'^4}\left[1-2\left(\frac{r^2}{r'^2}-\frac{2r}{r'}\cos\theta\right)\right] \quad \text{for} \quad \vec{r} \le \vec{r}' . \tag{29}$$

THE MATHEMATICAL SOLUTION OF THE *B* AND *H* FIELDS FOR THE $1/|\vec{r}-\vec{r}'|^3$ SOURCE

We substitute equations (28) and (29) into equations (26) and (27) and the result into the source terms of equations (23), (24), and (25). By applying the boundary conditions for the *B* and *H* fields to the resulting equations and setting the coefficients of the powers of the sines and cosines equal to zero, we generate the 10 equations and 10 unknowns, which are solved to obtain the coefficients of expansion. These coefficients are then substituted back into equations (1), (2), and (3) to yield a numerical solution.

The boundary at $r = b$ has two cases: $\vec{r} \le \vec{r}'$ and $\vec{r} \ge \vec{r}'$. For the continuity of B_r, the coefficients of $(\cos\theta)^0$ are

$$\frac{B_0}{r'^3}\left(1-\frac{2b^2}{r'^2}\right)-\frac{\alpha_0}{b^2}+\frac{3}{2}\left(\frac{\alpha_2}{b^4}\right)=-\mu\gamma_0\left(\frac{1}{b^2}\right)-\mu\beta_2 b+\frac{3}{2}\left(\frac{\mu\gamma_2}{b^4}\right) \quad \text{for} \quad \vec{r} \le \vec{r}' \tag{30}$$

and

$$\frac{B_0}{b^3}\left(1-\frac{2r'^2}{b^2}\right)-\frac{\alpha_0}{b^2}+\frac{3}{2}\left(\frac{\alpha_2}{b^4}\right)=-\mu\gamma_0\left(\frac{1}{b^2}\right)-\mu\beta_2 b+\frac{3}{2}\left(\frac{\mu\gamma_2}{b^4}\right) \quad \text{for} \quad \vec{r} \ge \vec{r}' \ ; \tag{31}$$

the coefficients of $(\cos\theta)^1$ are

$$\frac{B_0}{r'^4}\left(\frac{4b^2}{r'}-b+\frac{2b^2}{r'}\right)-\frac{2\alpha_1}{b^3}=\mu\beta_1-\mu\gamma_1\left(\frac{2}{b^3}\right) \quad \text{for} \quad \vec{r} \le \vec{r}' \tag{32}$$

and

$$\frac{B_0}{b^4}\left(3r'+\frac{r'^3}{b^2}\right)-\frac{2\alpha_1}{b^3}=\mu\beta_1-\mu\gamma_1\left(\frac{2}{b^3}\right) \quad \text{for} \quad \vec{r} \ge \vec{r}' \ ; \tag{33}$$

and the coefficients of $(\cos\theta)^2$ are

$$-\frac{B_0}{r'^4}(4b)-\frac{9\alpha_2}{2b^4}-=3\frac{\mu}{2}\left(2\beta_2 b-\frac{3\gamma_2}{b^4}\right) \quad \text{for } \vec{r}\le\vec{r}' \tag{34}$$

and

$$-\frac{B_0}{b^5}\left(4r'^2\right)-\frac{9\alpha_2}{2b^4}-=3\frac{\mu}{2}\left(2\beta_2 b-\frac{3\gamma_2}{b^4}\right) \quad \text{for } \vec{r}\ge\vec{r}' . \tag{35}$$

For the continuity of H_θ at $r = b$, there are no terms in the coefficient for $(\cos\theta)^0$. For the coefficients of $(\sin)^1$,

$$\frac{B_0}{r'^4}\left(r'-\frac{2b^2}{r'}\right)-\frac{\alpha_1}{b^2}=-\left(\beta_1 b+\gamma_1\frac{1}{b^2}\right) \quad \text{for } \vec{r}\le\vec{r}' \tag{36}$$

and

$$\frac{B_0}{r^4}\left(r'-\frac{2r'^3}{b^2}\right)-\frac{\alpha_1}{b^2}=-\left(\beta_1 b+\gamma_1\frac{1}{b^2}\right) \quad \text{for } \vec{r}\ge\vec{r}' . \tag{37}$$

For the coefficients of $(\sin\theta\cos\theta)^1$,

$$\frac{B_0}{r'^4}(4b)+3\frac{\alpha_2}{b^3}=-\frac{3}{\gamma_2}\left(\frac{1}{b^3}\right) \quad \text{for } \vec{r}\le\vec{r}' \tag{38}$$

and

$$\frac{B_0}{b^5}\left(4r'^2\right)+3\frac{\alpha_2}{b^3}=-\frac{3}{\gamma_2}\left(\frac{1}{b^3}\right) \quad \text{for } \vec{r}\ge\vec{r}' . \tag{39}$$

For the continuity of B_r at $r=a$, the coefficient of $(\cos\theta)^0$ is

$$-\frac{\gamma_0}{a^2}\mu+\frac{1}{2}\left(2\beta_2 a-\gamma_2\frac{3}{a^4}\right)=0 ; \tag{40}$$

the coefficient of $(\cos\theta)^1$ is

$$\beta_1-\gamma_1\frac{2}{a^3}=\delta_1 ; \tag{41}$$

and the coefficient of $(\cos\theta)^2$ is

$$\frac{3}{2}\left(2\beta_2 a - \gamma_2 \frac{3}{a^4}\right) = 3\delta_2 a \ . \tag{42}$$

For the continuity of H_θ at $r = a$, the coefficient of $(\sin)^1$ is

$$-\left(\beta_1 a + \gamma_1 \frac{1}{a^2}\right) = -\delta_1 a , \tag{43}$$

and the coefficient of $(\sin\theta\cos\theta)^1$ is

$$\beta_2 a^2 + \gamma_2 \frac{1}{a^3} = \delta_2 a^2 \ . \tag{44}$$

Solving the 10 equations and 10 unknowns with μ complex, which represents a composite material, yields the magnetostatic solution (calculated with *Mathematica*[7] as shown in the appendix).

5. CONCLUSIONS

This research develops a new technique to model fields from transformers and power supplies at low frequencies in the nearfield region shielded by cylindrical shells of composite materials. The first step in the process formulates the magnetostatic problem by solving Laplace's equation in terms of binomial and Legendre polynomial expansions. With the solution to the magnetostatic problem used as a basis, Ampere's law generates an $\bar{E}$ field with complex permittivity. This $\bar{E}$ field becomes the basis for a new $\bar{H}$ field in Faraday's law. A perturbation expansion is then generated for the $\bar{B}$, $\bar{H}$, $\bar{E}$, and $\bar{D}$ fields. The boundary conditions are applied to these new fields, and the coefficients of expansion are calculated. The intensities and Poynting vectors are also obtained in the various regions. Plots of these intensities versus the real and imaginary parts of the permeability and the real and imaginary parts of the permittivity will be provided in a future report.

6. REFERENCES

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APPENDIX

THE USE OF *MATHEMATICA* TO OBTAIN A SPECIFIC SOLUTION

■ Description

This *Mathematica* program solves the magnetostatic problem of a power supply interacting in the nearfield with a cylindrical shell of composite material that has complex permeability.

■ Defining Terms

```
u=1/r^2   (1-rPrime^2/r^2-(2*rPrime/r)*Cos[theta])

             2
     rPrime     2 rPrime Cos[theta]
1 - ------- - -------------------
        2              r
       r
---------------------------------
                2
               r

v=(r+rPrime*Cos[theta])

r + rPrime Cos[theta]

w=rPrime*Sin[theta]

rPrime Sin[theta]
```

The number of terms in the Legendre Polynomials

```
n:=2
```

Φ outside the cylinder

```
Øm:=Sum[(alpha[l]/r^(l+1))*LegendreP[l,Cos[theta]],{l,0,n}]
```

Φ within the material of the cylinder

```
Øq:=Sum[(beta[l]*r^l  +  (gamma[l]/r^(l+1)))  LegendreP[l,Cos[theta]],  {l,0,n}]
```

Φ inside the cylinder

```
Øp:=Sum[delta[l]*r^l*LegendreP[l,Cos[theta]],  {l,0,n}]
```

■ Setting the Boundary conditions

```
FourEqn={
Bo*(v*u)-D[Øm,  r]-(-μ*D[Øq,  r])==0/.r->b,
Bo*(w*u)-(1/r)*D[Øm,  theta]-(-(1/r)*D[Øq,  theta])==0/.r->b,
-D[Øq,  r]-(-D[Øp,  r])==0/.r->a,
(-1/r)*D[Øq,  theta]-((-1/r)*D[Øp,  theta])==0/.r->a
}
```

```
 alpha[0]    2 alpha[1] Cos[theta]
{-------- + --------------------- +
     2                3
    b                b

                                                 2
                                           rPrime     2 rPrime Cos[theta]
   Bo (b + rPrime Cos[theta]) (1 - ------- - -------------------)
                                              2                 b
                                             b
   -------------------------------------------------------------- +
                                     2
                                    b

                                    2
  3 alpha[2] (-1 + 3 Cos[theta] )
  ------------------------------- + μ
                   4
                2 b

      gamma[0]                               2 gamma[1]
   (-(--------) + Cos[theta] (beta[1] - ----------) +
          2                                  3
         b                                  b
```

```
                        2                     3 gamma[2]
   (-1 + 3 Cos[theta] ) (2 b beta[2] - ----------)
                                                 4
                                                b
   ----------------------------------------------) == 0,
                          2

                   2
              rPrime     2 rPrime Cos[theta]
Bo rPrime (1 - ------- - -------------------) Sin[theta]
                  2               b
                 b
---------------------------------------------------------- -
                          2
                         b

    alpha[1] Sin[theta]     3 alpha[2] Cos[theta] Sin[theta]
  -(-------------------) - --------------------------------
                2                         3
               b                         b
  ---------------------------------------------------------- +
                             b

                  gamma[1]                                    2            gamma[2]
  -((b beta[1] + --------) Sin[theta]) - 3 Cos[theta] (b  beta[2] + --------) Sin[theta]
                      2                                                    3
                     b                                                    b
  ---------------------------------------------------------------------------------------- == 0
                                          b

                                                    2              gamma[0]
   Cos[theta] delta[1] + a (-1 + 3 Cos[theta] ) delta[2] + -------- -
                                                                  2
                                                                 a
```

```
                                                  2               3 gamma[2]
                                 (-1 + 3 Cos[theta] ) (2 a beta[2] - ----------)
                                                                          4
                  2 gamma[1]                                             a
 Cos[theta] (beta[1] - ----------) - ------------------------------------------------ == 0,
                           3                           2
                          a

                          2
-(a delta[1] Sin[theta]) - 3 a  Cos[theta] delta[2] Sin[theta]
-------------------------------------------------------------- -
                          a

                gamma[1]                              2              gamma[2]
 -((a beta[1] + --------) Sin[theta]) - 3 Cos[theta] (a  beta[2] + --------) Sin[theta]
                    2                                                  3
                   a                                                  a
 ------------------------------------------------------------------------------------ == 0
                                      a

  }
```

Simplify the result from above without using Trigonometric identities

FourEqn1=Simplify[FourEqn,Trig->False]

```
 alpha[0]   2 alpha[1] Cos[theta]
{-------- + --------------------- +
     2                3
    b                b

                                    2          2
   Bo (b + rPrime Cos[theta]) (b  - rPrime  - 2 b rPrime Cos[theta])
   ----------------------------------------------------------------- +
                                    4
                                   b
```

```
                                2
 3 alpha[2] (-1 + 3 Cos[theta] )
 ------------------------------- + μ
                  4
               2 b

    gamma[0]                                2 gamma[1]
  (-(--------) + Cos[theta] (beta[1] - ----------) +
        2                                    3
       b                                    b

                       2                    3 gamma[2]
    (-1 + 3 Cos[theta] ) (2 b beta[2] - ----------)
                                              4
                                             b
    ----------------------------------------------) == 0,
                          2

   2                     3          4                      2
((b  Bo rPrime - Bo rPrime  + b alpha[1] - b  beta[1] - 2 b Bo rPrime  Cos[theta] +

                                  5
     3 alpha[2] Cos[theta] - 3 b  beta[2] Cos[theta] - b gamma[1] - 3 Cos[theta] gamma[2])

                     4                                                2                gamma[0]
    Sin[theta]) / b   == 0, Cos[theta] delta[1] + a (-1 + 3 Cos[theta] ) delta[2] + -------- -
                                                                                           2
                                                                                          a

                                                    2                    3 gamma[2]
                                  (-1 + 3 Cos[theta] ) (2 a beta[2] - ----------)
                                                                             4
                        2 gamma[1]                                          a
 Cos[theta] (beta[1] - ----------) - ------------------------------------------------ == 0,
                             3                                2
                            a
```

```
  4               5                          4              5
((a  beta[1] + 3 a  beta[2] Cos[theta] - a  delta[1] - 3 a  Cos[theta] delta[2] +

                                                           4
     a gamma[1] + 3 Cos[theta] gamma[2]) Sin[theta]) / a  == 0}
```

Find all the coefficients of Sin(θ) and Cos(θ)

`frst=Table[CoefficientList[First[FourEqn1[[i]]],{Cos[theta],Sin[theta]}],   {i,1,4}]`

```
                   2
   Bo    Bo rPrime     alpha[0]    3 alpha[2]                  μ gamma[0]   3 μ gamma[2]
{{{-- - ---------- + -------- - ---------- - μ b beta[2] - ---------- + ------------},
   b         3            2            4                         2              4
            b            b          2 b                         b            2 b

                               3
    Bo rPrime     Bo rPrime     2 alpha[1]                   2 gamma[1]
  {-(---------) - ---------- + ---------- + μ (beta[1] - ----------)},
         2             4             3                          3
        b             b             b                          b

                 2
   -2 Bo rPrime     9 alpha[2]                   9 μ gamma[2]
  {------------- + ---------- + 3 μ b beta[2] - ------------}},
         3               4                             4
        b             2 b                           2 b

                             3
      Bo rPrime    Bo rPrime     alpha[1]               gamma[1]
 {{0, --------- - ---------- + -------- - beta[1] - --------},
           2            4            3                    3
          b            b            b                    b
```

```
                    2
      -2 Bo rPrime     3 alpha[2]                 3 gamma[2]
{0, ------------- + ---------- - 3 b beta[2] - ----------}},
            3              4                          4
           b              b                          b

                                 gamma[0]   3 gamma[2]                                    2 gamma[1]
{{a beta[2] - a delta[2] + -------- - ----------}, {-beta[1] + delta[1] + ----------},
                                    2             4                                          3
                                   a           2 a                                          a

                                9 gamma[2]
{-3 a beta[2] + 3 a delta[2] + ----------}},
                                    4
                                 2 a

                            gamma[1]                                        3 gamma[2]
{{0, beta[1] - delta[1] + --------}, {0, 3 a beta[2] - 3 a delta[2] + ----------}}}
                               3                                                4
                              a                                                a
```

`Length[Flatten[frst]]`

14

Remove all the coefficients that are equal to 0

`frst=Complement[Flatten[frst], {0}]`

```
                       gamma[1]                          2 gamma[1]
{beta[1] - delta[1] + --------, -beta[1] + delta[1] + ----------,
                          3                                  3
                         a                                  a

                          3
 Bo rPrime   Bo rPrime     alpha[1]                gamma[1]
 --------- - ----------- + -------- - beta[1] - --------,
     2            4            3                    3
    b            b            b                    b
```

$$-\left(\frac{Bo\ rPrime}{b^{2}}\right) - \frac{Bo\ rPrime^{3}}{b^{4}} + \frac{2\ alpha[1]}{b^{3}} + \mu\ \left(beta[1] - \frac{2\ gamma[1]}{b^{3}}\right),$$

$$a\ beta[2] - a\ delta[2] + \frac{gamma[0]}{a^{2}} - \frac{3\ gamma[2]}{2\ a^{4}},\ 3\ a\ beta[2] - 3\ a\ delta[2] + \frac{3\ gamma[2]}{a^{4}},$$

$$-3\ a\ beta[2] + 3\ a\ delta[2] + \frac{9\ gamma[2]}{2\ a^{4}},$$

$$\frac{-2\ Bo\ rPrime^{2}}{b^{3}} + \frac{3\ alpha[2]}{b^{4}} - 3\ b\ beta[2] - \frac{3\ gamma[2]}{b^{4}},$$

$$\frac{-2\ Bo\ rPrime^{2}}{b^{3}} + \frac{9\ alpha[2]}{2\ b^{4}} + 3\ \mu\ b\ beta[2] - \frac{9\ \mu\ gamma[2]}{2\ b^{4}},$$

$$\frac{Bo}{b} - \frac{Bo\ rPrime^{2}}{b^{3}} + \frac{alpha[0]}{b^{2}} - \frac{3\ alpha[2]}{2\ b^{4}} - \mu\ b\ beta[2] - \frac{\mu\ gamma[0]}{b^{2}} + \frac{3\ \mu\ gamma[2]}{2\ b^{4}}\}$$

Length[Flatten[frst]]

Set all the coefficients equal to 0

```
Equations=Table[Simplify[Equal[frst[[i]],0]],{i,1,Length[frst]}]
```

```
TableForm[Equations]
```

```
                          gamma[1]
beta[1] - delta[1] + -------- == 0
                              3
                             a

                            2 gamma[1]
-beta[1] + delta[1] + ---------- == 0
                                3
                               a

 2                      3                   4
b  Bo rPrime - Bo rPrime  + b alpha[1] - b  beta[1] - b gamma[1]
---------------------------------------------------------------- == 0
                                4
                               b

  2                      3                        4
-(b  Bo rPrime) - Bo rPrime  + 2 b alpha[1] + μ b  beta[1] - 2 μ b gamma[1]
--------------------------------------------------------------------------- == 0
                                     4
                                    b

                         gamma[0]   3 gamma[2]
a beta[2] - a delta[2] + -------- - ---------- == 0
                             2            4
                            a          2 a

                             3 gamma[2]
3 a beta[2] - 3 a delta[2] + ---------- == 0
                                   4
                                  a
```

```
                                9 gamma[2]
-3 a beta[2] + 3 a delta[2] + ---------- == 0
                                     4
                                  2 a

                   2               5
-2 b Bo rPrime  + 3 alpha[2] - 3 b  beta[2] - 3 gamma[2]
-------------------------------------------------------- == 0
                                4
                               b

                   2                 5
-4 b Bo rPrime  + 9 alpha[2] + 6 µ b  beta[2] - 9 µ gamma[2]
------------------------------------------------------------ == 0
                                   4
                                2 b

    3                     2      2                             5                2
(2 b  Bo - 2 b Bo rPrime  + 2 b  alpha[0] - 3 alpha[2] - 2 µ b  beta[2] - 2 µ b  gamma[0] +

                         4
     3 µ gamma[2]) / (2 b ) == 0
```

Define all the possible unknowns

`UnKnowns=Flatten[Table[{alpha[i],beta[i],gamma[i],delta[i]},  {i,0,n}]]`

```
{alpha[0], beta[0], gamma[0], delta[0], alpha[1], beta[1], gamma[1], delta[1], alpha[2],

  beta[2], gamma[2], delta[2]}
```

■ Solve the Equations for the Unknowns

Solve the equations for the unknowns

`Coeffs=Solve[Equations,  UnKnowns]`

`TableForm[Flatten[Coeffs]]`

Simplify the final equations for the coefficients of the Legendre Polynomials

Simplify[Coeffs]

```
                       2              2
                Bo (-3 b  + 5 rPrime )
{{alpha[0] -> ----------------------, gamma[0] -> 0,
                        3 b

                          2       2           2            2
                Bo rPrime (b  - μ b  + rPrime  + μ rPrime )
   alpha[1] -> -------------------------------------------,
                                2 b + μ b

                             2        2
                Bo rPrime (3 b  - rPrime )                  4 (1 + μ) b Bo rPrime
   delta[1] -> --------------------------, alpha[2] -> ----------------------,
                                4                              9 + 6 μ
                      (2 + μ) b

                              2
                -2 Bo rPrime                     Bo rPrime (3 b  - rPrime )
   delta[2] -> --------------, beta[1] -> --------------------------, gamma[1] -> 0,
                             4                                    4
                3 (3 + 2 μ) b                           (2 + μ) b

                            2
               -2 Bo rPrime
   beta[2] -> --------------, gamma[2] -> 0}}
                             4
               3 (3 + 2 μ) b
```

■ Solving for H

```
Needs["Calculus`VectorAnalysis`"]
```

The Field outside the Cylinder:

```
H={0,(Bo/Abs[r-rPrime]*(r-rPrime/Abs[r-rPrime])),0}-
Grad[Sum[alpha[i]/r^i*LegendreP[i,  Cos[theta]],  {i,  0,2}],Cylindrical]
```

```
                                                    2
 alpha[1] Cos[theta]   alpha[2] (-1 + 3 Cos[theta] )
{------------------- + -----------------------------,
          2                         3
         r                         r

                                   alpha[1] Sin[theta]    3 alpha[2] Cos[theta] Sin[theta]
             rPrime              -(-------------------) - --------------------------------
  Bo (r - ---------------)                 r                             2
           Abs[r - rPrime]                                              r
  ------------------------ - ------------------------------------------------------------, 0}
      Abs[r - rPrime]                                     r
```

Substitute the Coefficients

```
H/.Coeffs
```

```
                   2        2           2            2
  Bo rPrime (b   - μ b   + rPrime   + μ rPrime ) Cos[theta]
{{------------------------------------------------------ +
                                      2
                       (2 b + μ b) r

                      2                   2
    4 (1 + μ) b Bo rPrime   (-1 + 3 Cos[theta] )
    ----------------------------------------------,
                            3
                 (9 + 6 μ) r

               rPrime
   Bo (r - ---------------)
           Abs[r - rPrime]         Bo rPrime (b   - μ b   + rPrime   + μ rPrime ) Sin[theta]
   ------------------------ - (-(----------------------------------------------------------) -
        Abs[r - rPrime]                                 (2 b + μ b) r

                               2
       12 (1 + μ) b Bo rPrime  Cos[theta] Sin[theta]
       ----------------------------------------------) / r, 0}}
                                      2
                           (9 + 6 μ) r
```

Along the axis:

```
%/.theta->0
```

```
                         2                 2       2          2            2
  8 (1 + μ) b Bo rPrime     Bo rPrime (b   - μ b   + rPrime   + μ rPrime )
{{--------------------- + ------------------------------------------,
                  3                                   2
       (9 + 6 μ) r                       (2 b + μ b) r

               rPrime
   Bo (r - ---------------)
```

```
          Abs[r - rPrime]
------------------------, 0}}
     Abs[r - rPrime]
```

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